

① $S_0^1 \rightarrow$ LU decomposition

$$\begin{aligned} x + 2y + z &= 4 \\ 2x - y + 3z &= 2 \\ -x + y - z &= -1 \end{aligned}$$

① LU: $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$

$$A = LU$$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ -1 & 1 & -1 \end{bmatrix}$$

②

$$(1,1) \quad l_{11} \cdot 1 = 1$$

$$(2,1) \quad l_{21} \cdot 1 = 2$$

$$(3,1) \quad l_{31} \cdot 1 = -1$$

$$(1,2) \quad l_{11} u_{12} = 2$$

$$u_{12} = 2$$

$$(2,2) \quad l_{21} u_{12} + l_{22} = -1$$

$$(2)(2) + l_{22} = -1$$

$$l_{22} = -5$$

$$(3,2) \quad l_{31} u_{12} + l_{32} = 1$$

$$(-1)(2) + l_{32} = 1$$

$$l_{32} = 3$$

$$(1,3) \quad l_{11} u_{13} = 1$$

$$u_{13} = 1$$

$$(2,3) \quad l_{21} u_{13} + l_{22} u_{23} = 3$$

$$(2)(1) + (-5)u_{23} = 3$$

$$u_{23} = \frac{1}{5}$$

$$(3,3) \quad l_{31} u_{13} + l_{32} u_{23} + l_{33} = -1$$

$$(-1)(1) + (3)\left(\frac{1}{5}\right) + l_{33} = -1$$

$$l_{33} = \frac{10}{5}$$

③ $\text{หาค่าที่หาได้มาแล้ว ;}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -5 & 0 \\ -1 & 3 & \frac{10}{5} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

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$$L\vec{U}\vec{x} = \vec{b}$$

$$L\vec{y} = \vec{b}; \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 5 & 0 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

$$y_1 = 4 \quad - \textcircled{1}$$

$$2y_1 - 5y_2 = 2 \quad - \textcircled{2}$$

mu y_1 ;

$$2(4) - 5y_2 = 2$$

$$y_2 = \frac{6}{5}$$

$$-y_1 + 3y_2 + \frac{3}{5}y_3 = -1 \quad - \textcircled{3}$$

mu y_1, y_2 ;

$$-4 + 3\left(\frac{6}{5}\right) + \frac{3}{5}y_3 = -1$$

$$y_3 = -1$$

$$\therefore \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{6}{5} \\ -1 \end{bmatrix}$$

$$U\vec{x} = \vec{y}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{6}{5} \\ -1 \end{bmatrix}$$

$$x + 2y + z = 4 \quad - \textcircled{1}$$

$$y - \frac{1}{5}z = \frac{6}{5} \quad - \textcircled{2}$$

$$z = -1 \quad - \textcircled{3}$$

mu z in $\textcircled{2}$;

$$y - \frac{1}{5}(-1) = \frac{6}{5}$$

$$y = 1$$

mu y, z in $\textcircled{1}$; $x + 2(1) + (-1) = 4$

$$x = 3$$

$$\therefore x = 3, y = 1, z = -1$$

② Solⁿ → Gaussian

$$\begin{aligned}x + 2y + z &= 4 \\ 2x - y + 3z &= 2 \\ -x + y - z &= -1\end{aligned}$$

①

Gaussian ;

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -1 & 3 & 2 \\ -1 & 1 & -1 & -1 \end{array} \right]$$

$R_2 \rightarrow R_2 - 2R_1$;

$R_3 \rightarrow R_3 + R_1$;

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & 1 & -6 \\ 0 & 3 & 0 & 3 \end{array} \right]$$

② \Rightarrow $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$

$$x + 2y + z = 4 \quad \text{--- ①}$$

$$-5y + z = -6 \quad \text{--- ②}$$

$$3y = 3 \quad \text{--- ③}$$

u y ;

$$y = 1 \quad \#$$

u z ;

$$-5 + z = -6$$

$$z = -1 \quad \#$$

u x ;

$$-x + 1 + 1 = -1$$

$$x = 3 \quad \#$$

$$\therefore x = 3, y = 1, z = -1$$

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จงหาคำของระบบสมการต่อไปนี้

$$2x + y - z = 0$$

$$x - 5y - 2z = 1$$

$$x - y - 3z = -6$$

① Gaussidn :

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & -5 & -2 & 1 \\ 1 & -1 & -3 & -6 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow R_2 - \frac{1}{2}R_1 ; \\ R_3 \rightarrow R_3 - \frac{1}{2}R_1 ; \end{array} \left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & \frac{-11}{2} & \frac{-3}{2} & 1 \\ 0 & \frac{-3}{2} & \frac{-5}{2} & -6 \end{array} \right]$$

$$R_3 \rightarrow R_3 - \frac{3}{11}R_2 ; \left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & \frac{-11}{2} & \frac{-3}{2} & 1 \\ 0 & 0 & \frac{-23}{11} & \frac{-19}{11} \end{array} \right]$$

② ใช้เมทริกซ์ผกผัน

$$2x + y - z = 0 \quad \text{--- ①}$$

$$\frac{-11y}{2} - \frac{3z}{2} = 1 \quad \text{--- ②}$$

$$\frac{-23z}{11} = \frac{-69}{11} \quad \text{--- ③}$$

$$z = 3$$

ใช้ y ; แทน z ใน ② ;

$$\frac{-11y}{2} - \frac{3}{2}(3) = 1$$

$$y = \frac{11}{2} \cdot \frac{-2}{11}$$

$$y = -1$$

ใช้ x ; แทน y, z ใน ① ;

$$2x - 1 - 3 = 0$$

$$2x = 4$$

$$x = 2$$

$$\therefore x = 2, y = -1, z = 3$$